

# Error vs Distortion

The word *error* comes from the Latin *errare* which means to be mistaken or incorrect; make a mistake. Synonyms of the verb to err are be wrong, be in error, be mistaken, mistake, make a blunder, blunder, be incorrect, be inaccurate, misjudge, miscalculate, get things/something/ it wrong, bark up the wrong tree, get the wrong end of the stick, be wide of the mark. It is not difficult to see that the word has many meanings but always with a negative connotation.

The famous Mercator made an error or mistake when he displayed the town Zagreb in his map *Sclavonia, Croatia, Bosnia cum Dalmatiae parte*. Zagreb is actually located in two different places on the map, appearing as both *Agram* and *Zagrabia*.

I made mistakes, typos or typographical errors in formulas for the computation of area of ellipsoidal trapezium (Lapaine 2017). Instead of

$$dP = \sqrt{EF - G^2} d\varphi d\lambda = MN \cos \varphi d\varphi d\lambda$$

and

$$P = \int_{\lambda_1}^{\lambda_2} \int_{\varphi_1}^{\varphi_2} MN \cos \varphi d\varphi d\lambda = \frac{b^2}{2} (\lambda_2 - \lambda_1) \cdot$$

$$\frac{\sin \varphi}{1 - e^2 \sin^2 \varphi} - \frac{1}{2e} \ln \frac{1 - e \sin \varphi}{1 + e \sin \varphi} \Big|_{\varphi_1}^{\varphi_2}$$

as it has been printed, it should be

$$dP = \sqrt{EG - F^2} d\varphi d\lambda = MN \cos \varphi d\varphi d\lambda$$

and

$$P = \int_{\lambda_1}^{\lambda_2} \int_{\varphi_1}^{\varphi_2} MN \cos \varphi d\varphi d\lambda = \frac{b^2}{2} (\lambda_2 - \lambda_1) \cdot \frac{\sin \varphi}{1 - e^2 \sin^2 \varphi} - \frac{1}{2e} \ln \frac{1 - e \sin \varphi}{1 + e \sin \varphi} \Big|_{\varphi_1}^{\varphi_2}$$

The word *distortion* comes from the Latin *distorsio*  $\approx$  *distorquere*: which means twisted, deformed, misshapen, misrepresented, altered. The synonyms are twist, warp, contort, bend, buckle, deform, malform, misshape, disfigure, mangle, wrench, wring, wrest. Distortion is the alteration of the original shape or other characteristic of something and has no negative connotation in general.

The Mercator projection, for example, distorts by exaggerating the size of regions at high latitude. In fact, all map projections involve distortion of areas, angles, and/or distances. The types of distortion can be controlled to preserve specific characteristics, but map projections must distort other characteristics of the object represented. The main problem in cartography is that it is not possible to map a spherical or ellipsoidal surface into a plane without distortions. Euler first proved as early as 1772 that a sphere cannot be mapped into a plane without distortions.

Although some authors, such as Airy (1861), used the term error, they actually meant distortion (Lapaine 2016). Airy named his projection the Projection by Balance of Errors. In his paper he used terms such as evil or

misrepresentation. He also uses the term distortion, but not in its present meaning. In my opinion, it is not adequate to use the term *error* in map projections because error usually means something wrong, not good or even bad.

As I had explained (Lapaine 2016), Airy made an error or a mistake in solving the problem of the Calculus of Variations. This error is something completely different from "errors" in his projection. In order to distinguish a mistake (*errare humanum est*) from inevitable or unavoidable distortions in map projections, I propose not using the term error in map projection theory. We must be aware that distortions are immanent in any map projection, we must be able to control the amount of distortions in any map, and we need to serve as educators to people who are not aware of the properties of map projections and who believe everything represented on a map, including its mathematical base, is without distortions. Instead of Projection by Balance of Errors, it would be better to refer to it as Projection by Balance of Distortions.

Thus, errors and distortions are two fundamentally different concepts in cartography. Each map is a distorted representation. But it is essential that any distortion on the map can be mathematically removed. If we measure on a map and do not take distortion into account, we will make an error!

Miljenko Lapaine ■

## References / Literatura

- Airy G B (1861) Explanation of a projection by balance of errors for maps applying to a very large extent of the Earth's surface; and comparison of this projection with other projections: *London, Edinburgh, and Dublin Philosophical Magazine*, series 4, v. 22, no. 149, p. 409–421. [Presents his minimum error azimuthal projection. See correction by James and Clarke (1862).]
- Biernacki F (1949) *Teoria odwzorowań powierzchni dla geodetów i kartografów*. Warsaw, Główny Urząd Pomiarów Kraju, Prace Geodezyjnego Instytutu Naukowo-Badawczego, no. 4. Polish. Translated into English as *Theory of representation of surfaces for surveyors and cartographers*: U.S. Dept. of Commerce, 1965
- Euler L (1777) De representatione superficiei sphaericae super plano, *Acta Academiae Scientiarum Imperialis Petropolitanae*. Translated into German in: Drei Abhandlungen über Kartenprojektion, *Ostwald's Klassiker der exakten*

# Pogreška ili distorzija

Riječ pogreška označava netočnu ili lošu ideju ili mišljenje, ono što je napravljeno loše, netočno ili nekorrektno. Riječi istog ili sličnog značenja su greška, zabluda, zabuna, nespornost, propust, gubitak, promašaj, gaf, omaška, previd. Nije teško uočiti da riječ pogreška ima više značenja, no uvijek s negativnom konotacijom.

Slavni Mercator je pogriješio kad je na svojoj karti *Sclavonia, Croatia, Bosnia cum Dalmatiae parte* prikazao Zagreb. Na toj se karti Zagreb pojavljuje na sva različita mjesta, kao *Agram* i kao *Zagrabia*.

Ja sam (Lapaine 2017) napravio zatipak, tipfeler ili slovnu pogrešku u formulama za računanje površine elipsoidnog trapeza. Umjesto

$$dP = \sqrt{EF - G^2} d\varphi d\lambda = MN \cos \varphi d\varphi d\lambda$$

i

$$P = \int_{\lambda_1, \varphi_1}^{\lambda_2, \varphi_2} MN \cos \varphi d\varphi d\lambda = \frac{b^2}{2} (\lambda_2 - \lambda_1) \cdot \frac{\sin \varphi}{1 - e^2 \sin^2 \varphi} - \frac{1}{2e} \ln \frac{1 - e \sin \varphi}{1 + e \sin \varphi} \Big|_{\varphi_1}^{\varphi_2}$$

kako je otisnuto, trebalo je biti

$$dP = \sqrt{EG - F^2} d\varphi d\lambda = MN \cos \varphi d\varphi d\lambda$$

i

$$P = \int_{\lambda_1, \varphi_1}^{\lambda_2, \varphi_2} MN \cos \varphi d\varphi d\lambda = \frac{b^2}{2} (\lambda_2 - \lambda_1) \cdot$$

*Wissenschaften*, no. 93, pp. 3–37, Leipzig, Wilhelm Engelmann, 1898. Translated into Polish in: Biernacki 1949. Translated into Russian: Ob izobrazhenii poverhnosti shara na ploskosti, *Trudy Petersburgskoy akademii nauk za 1777 g.*, Tom 1, pp. 107–132, in: L. Ejler: *Izbrannye kartograficheskie stat'i*, Tri stat'i po matematicheskoi kartografii, Izdatel'stvo geodezicheskoi literatury, 1959, Moscow. Translated into English in: Biernacki 1965. Translated into Croatian in: Lapaine 2014  
Lapaine M (2014) Kartografske projekcije i njihove deformacije (in Croatian), Peti

$$\frac{\sin \varphi}{1 - e^2 \sin^2 \varphi} - \frac{1}{2e} \ln \frac{1 - e \sin \varphi}{1 + e \sin \varphi} \Big|_{\varphi_1}^{\varphi_2}$$

Riječ *distorzija* dolazi iz latinskoga *distorsio*  $\approx$  *distorquere*: što znači uvijeno, izvrnuto, deformirano, pogrešno oblikovano, pogrešno prikazano, izobličeno, promijenjeno. Distorzija je promjena izvornog oblika ili neke druge karakteristike i nema općenito negativnu konotaciju.

Primjerice Mercatorova projekcija uveličava veličinu područja u većim geografskim širinama. Zapravo sve kartografske projekcije deformiraju površine, kutove i/ili udaljenosti. Neke distorzije na kartama mogu se kontrolirati kako bi se sačuvala određene karakteristike, ali će kartografska projekcija deformirati druge karakteristike prikazanog objekta. Glavni problem u kartografiji je nemogućnost preslikavanja sferne ili elipsoidne plohe u ravninu bez distorzije. Euler je prvi još 1772. godine dokazao da se sfera ne može preslikati u ravninu bez distorzija.

Premda neki autori, kao što je npr. Airy (1861), rabe naziv pogreška (*error*), oni zapravo misle na distorziju (Lapaine 2016). Airy je dao ime svojoj projekciji Projekcija uz balans pogrešaka (*Projection by Balance of Errors*). U svojem članku on upotrebljava i nazive vrag (*evil*) ili pogrešan prikaz (*misrepresentation*).

Upotrebljava i riječ distorzija (*distortion*), ali ne u današnjem značenju. Smatram da naziv pogreška u kartografskim projekcijama nije adekvatan jer pogreška obično znači nešto pogrešno, ne lijepo ili čak loše.

Kao što sam već objasnio (Lapaine 2016), Airy je pogriješio pri rješavanju problema računa varijacija. Takva pogreška je nešto sasvim različito od "pogrešaka" zbog projekcije. Kako bismo razlikovali pogrešku (*errare humanum est*) od neizbježnih distorzija u kartografskim projekcijama, predlažem da se izraz pogreška ne koristi u teoriji kartografskih projekcija.

Moramo biti svjesni da su distorzije imanentne u bilo kojoj kartografskoj projekciji, moramo biti u mogućnosti kontrolirati vrstu i veličinu distorzije na bilo kojoj karti i moramo poučavati osobe koje nisu svjesne svojstava kartografskih projekcija i koje vjeruju da je sve prikazano na karti, uključujući matematičku osnovu, bez distorzija. Umjesto Projekcija uz balans pogrešaka, bolje bi bilo reći Projekcija uz balans distorzija.

Dakle, pogreške i distorzije su dva fundamentalno različita koncepta u kartografiji. Svaka karta je deformirani prikaz. No bitno je da se svaka distorzija na karti može matematički ukloniti. Ako mjerimo na karti, a ne uzmemo u obzir distorziju, učinit ćemo pogrešku!

hrvatski kongres o katastru, Zagreb, 8–9. 5. 2014., Zbornik radova, ISBN 978-953-97081-9-9, pp. 15–32

Lapaine M (2016) George Biddell Airy and His Contribution to Map Projections Theory, Proceedings, 6 th International Conference on Cartography and GIS, 13–17 June 2016, Albena, Bulgaria, ISSN: 1314-0604, Eds: Bandrova T., Konecny M., 334–342.

Lapaine M (2017) Basics of Geodesy for Map Projections, in: Lapaine, M., E. L. Usery (eds.): Choosing a Map Projection, Lecture Notes in Geoinformation and Cartography, Springer, 2017, 327–343

Viličić M, Lapaine M (2015) Research on the Reliability of Mercator's Map Sclavonia, Croatia, Bosnia cum Dalmatiae parte, in: Cartography – Maps Connecting the World, C. Robbi Sluter, C. B. Madureira Cruz, P. M. Leal de Menezes (Eds.), Springer International Publishing, Series: Publications of the International Cartographic Association (ICA), doi 10.1007/978-3-319-17738-0\_15, Print ISBN 978-3-319-17737-3, Online ISBN 978-3-319-17738-0, 223–233

Miljenko Lapaine ■