The Polar, Equatorial and Oblique Aspects Have Only Azimuthal Projections

In two papers (Lapaine, Frančula 2016; Frančula, Lapaine 2018) we defined azimuthal, conic, cylindrical and other groups of projections, followed by the aspects of these projections, using a graticule of pseudomeridians and pseudoparallels. The graticule is produced by imagining the rotation of a graticule of meridians and parallels in any other position. The graticule of pseudomeridians and pseudoparallels has the important property that, independent of the position of the axis of the projection, mapped into the plane it retains a characteristic shape for each group of projections and thus allows each to be defined unambiguously. The line which passes through the poles of the graticule of pseudomeridians and pseudoparallels is called the axis of the projection.

The *aspect of the projection* is the position of the axis of the projection in relation to the axis of rotation of the Earth's sphere. The aspect may be normal, transverse, or oblique.

In the *normal aspect*, the axis of the projection corresponds with the axis of the Earth's sphere and the graticule of pseudomeridians and pseudoparallels corresponds with the graticule of meridians and parallels.

In the *transverse aspect*, the axis of the projection is perpendicular to the axis of the Earth's sphere.

The *oblique aspect* is neither normal nor transverse.

It should be said that the literature on map projections often uses the terms normal, transverse and oblique projections instead of normal, transverse and oblique aspects.

The shape of the graticule of pseudomeridians and pseudoparallels in a group of map projections has the shape of the graticule of meridians and parallels in the normal aspect of that group of projections. This is the simplest shape of the graticule in that group of projections and all map equations are associated with it (Lapaine, Frančula 2016). For example, Figure 1 shows the azimuthal projection in the normal aspect, while Figure 2 shows the cylindrical projection in the normal aspect.

Kessler (2018, 14) criticised the term 'normal aspect' and thought that it was not defined in a unambiguous way. "For example, normal aspect was often defined according to the *typical* aspect in which a projection was shown (e.g., an azimuthal projection was typically shown as centered on a pole whereas a cylindrical projection was typically shown as aligned along the Equator). Thus, "normal aspect" was not consistently applied to one particular aspect but was dependent upon the projection class and could be easily misunderstood."

On page 19 he writes, "Adopting familiar terms such as "equatorial," "oblique," and "polar" to define a projection's aspect would make it clear to the student where the geographic center of the map is located."

Kessler's criticism that the normal aspect in the azimuthal projection (Figure 1) and the normal aspect in the cylindrical projection (Figure 2) are in fact two different aspects would be justified if the aspect of the projection could be unambiguously defined depending on whether the geographic pole is at the centre of the map (polar aspect), a point on the equator (equatorial aspect) or any point between them (oblique aspect). We have showed that this definition of aspects is not unambiguous (Lapaine, Frančula 2019). Here, we provide more proof that the division of aspects into polar, equatorial and oblique cannot be applied to any group of projections except azimuthal projections. Figure shows a map of the USA in the normal aspect conic projection.

If we divide aspects into polar, equatorial and oblique (Kessler, Battersby 2019), what is the aspect of the map in Figure 3? Since the centre of the map is not a geographical pole or the equator, the only answer that it is the oblique aspect. According to this means of defining aspect, all maps of the USA must be in the oblique aspect, which is simply not correct. Maps of the USA, like maps of most other areas, can be in all three aspects, normal, transverse and oblique. In Figure 3, the map of the USA is in the normal equal-area conic projection, and in Figure 4 in the transverse conformal cylindrical projection. For a map of any area it is necessary to select the most appropriate projection in the most appropriate aspect, according to the position of the area mapped on the Earth's sphere and its geometric properties (size, shape, direction). Since the USA extends across medium latitudes, the difference from north to south does not exceed 32°, and its west-east extent is greater than its south-north extent, normal conic projections are most suitable for maps of the USA (Lapaine, Frančula 2005).

In order to confirm the correctness of this understanding of aspects of a projection, we will cite the work of connoisseurs of map projections. Snyder (1987) writes on page 98, "Like other normal conics, the Albers Equal-Area Conic projection (fig. 20) has concentric arcs of circles for parallels and equally spaced radii as meridians". This is the shape of the graticule

Polarni, ekvatorski i kosi aspekt imaju samo azimutne projekcije

U dva rada (Lapaine, Frančula 2016; Frančula, Lapaine 2018) definirali smo azimutne, konusne, cilindrične i ostale skupine projekcija, a potom i aspekt tih projekcija, s pomoću mreže pseudomeridijana i pseudoparalela. Ta se mreža dobiva zamišljenom rotacijom mreže meridijana i paralela u bilo koji drugi položaj. Mreža pseudomeridijana i pseudoparalela ima to važno svojstvo da neovisno o položaju osi projekcije preslikana u ravninu zadržava karakteristični oblik za pojedinu skupinu projekcija i time omogućava jednoznačnu definiciju svake skupine projekcija. Pravac koji prolazi polovima mreže pseudomeridijana i pseudoparalela naziva se os projekcije.

Aspekt projekcije je položaj osi projekcije u odnosu na os rotacije Zemljine sfere. Aspekt može biti uspravni (normalni, regularni), poprečni ili kosi.

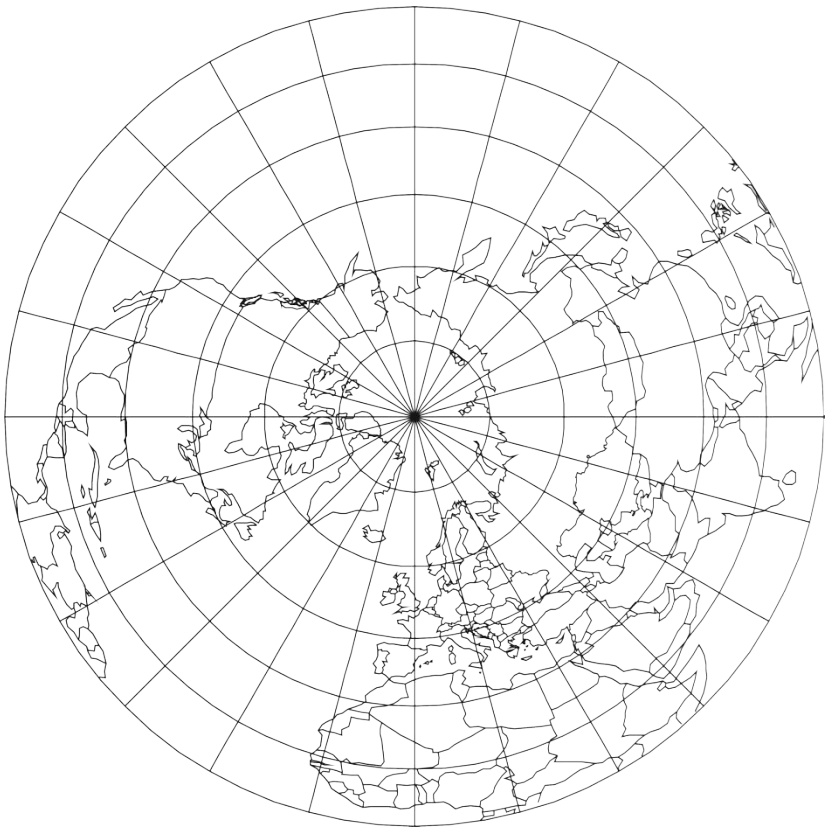
Uspravni aspekt je aspekt pri kojem se os projekcije podudara s osi Zemljine sfere, a mreža pseudomeridijana i pseudoparalela podudara se s mrežom meridijana i paralela.

Poprečni aspekt je aspekt pri kojem je os projekcije okomita na os Zemljine sfere.

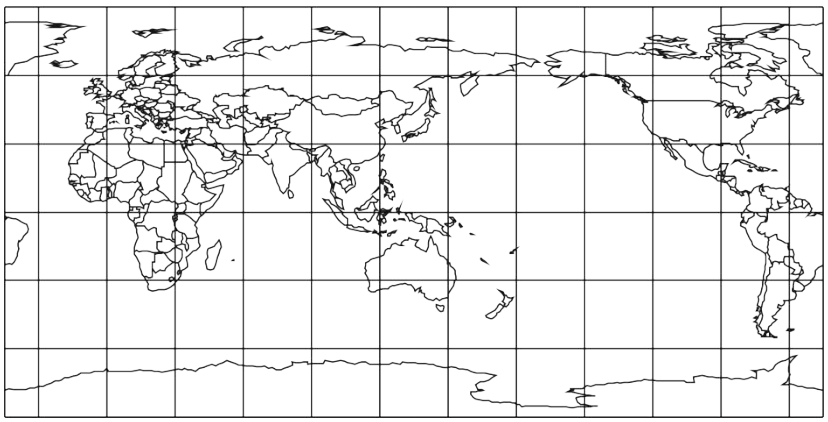
Kosi aspekt je aspekt koji nije ni uspravan ni poprečan.

Treba reći da se u literaturi o kartografskim projekcijama umjesto termina uspravni, poprečni i kosi aspekt neke projekcije češće rabe termini uspravna, poprečna i kosa projekcija.

Oblik mreže pseudomeridijana i pseudoparalela u nekoj skupini kartografskih projekcija ima mreža meridijana i paralela u uspravnom (normalnom) aspektu te skupine projekcija. To je najjednostavniji oblik te mreže u toj skupini projekcija i uz njega su vezane osnovne



Slika 1. Uspravni (normalni) aspekt Lambertove (ekvivalentne) azimutne projekcije.
Fig. 1 Normal aspect of conformal azimuthal projection.



Slika 2. Uspravni (normalni) aspekt ekvidistantne cilindrične projekcije.
Fig. 2 Normal aspect of equidistant cylindrical projection.

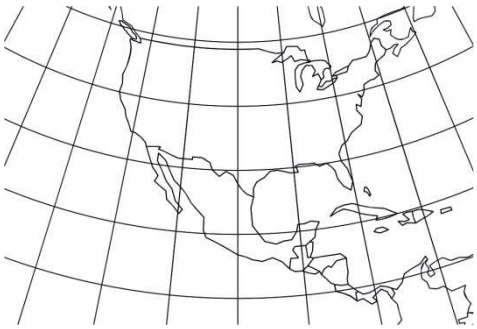


Fig. 3 Map of the USA in normal equal-area conic projection.
Slika 3. Karta SAD-a u uspravnoj ekvivalentnoj konusnoj projekciji.

Slika 4. Karta SAD-a u poprečnoj konformnoj cilindričnoj projekciji.
Fig. 4 Map of the USA in transverse conformal cylindrical projection.

of meridians and parallels in Figure 3 in this paper, therefore it is a normal conic projection.

On page 29 Snyder explains how the terms polar and equatorial aspect can only be applied to azimuthal projections. "If either true pole is at the center of an azimuthal map projection, the projection is called the *polar* aspect. If a point on the Equator is made the center, the projection is called the *equatorial* or, less often, *meridian* or *meridional* aspect. If some other point is central, the projection is the *oblique* or, occasionally, *horizon* aspect. For cylindrical and most other projections, such transformations are called *transverse* or *oblique*, depending on the angle of rotation."

In his book on shaping the mathematic basis of small-scale maps, Canter (2002) writes on page 73, "Traditionally three aspects of a map projection are distinguished: the normal (or direct) aspect, the transverse aspect, and the oblique aspect". In the glossary of terms *aspect of map projection* in various forms occupies 27 lines of text, while the terms *polar aspect* and *equatorial aspect* are absent.

We should also mention this. The normal aspect can be defined as the aspect in which the graticule of the meridians and parallels for a given group of projections has the simplest shape characteristic of the group. According, the maps in Figures 1, 2, 3, and 4 are not in the polar, equatorial and oblique aspects, but are in the normal (polar) aspect of azimuth projections (Figure 1), the normal aspect of cylindrical projections (Figure 2), the normal aspect of conic projections (Figure 3) and the transverse aspect of cylindrical projections (Figure 4).

Kessler's claim (2018) that the normal aspect is not unambiguously defined does not stand. According to the position of the axis of the projection in relation to the axis of rotation of the Earth's sphere, it is defined unambiguously for all important groups of projections. In the normal aspect, the axis of the projection corresponds to the axis of the rotation of the Earth's sphere (Frančula, Lapaine 2018). It is also unambiguously defined as the aspect in which the graticule of the meridians and parallels has the simplest shape in the given group of projections.

Finally, a question for those who advocate using the terms polar, equatorial and oblique aspects. In which of these three aspects is Mercator's transverse projection, which has many applications, and forms the mathematical basis of UTM?

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kartografske jednadžbe (Lapaine, Frančula 2016). Npr. na slici 1 je azimutna projekcija u normalnom aspektu, a na slici 2 cilindrična projekcija u normalom aspektu.

Kessler (2018, str. 14) kritizira termin normalni aspekt i smatra da nije jednoznačno definiran: „For example, normal aspect was often defined according to the *typical* aspect in which a projection was shown (e.g., an azimuthal projection was typically shown as centered on a pole whereas a cylindric projection was typically shown as aligned along the Equator). Thus, "normal aspect" was not consistently applied to one particular aspect but was dependent upon the projection class and could be easily misunderstood...“

Na str. 19 piše: „Adopting familiar terms such as "equatorial," "oblique," and "polar" to define a projection's aspect would make it clear to the student where the geographic center of the map is located“.

Kesslerova kritika da su normalni aspekt u azimutnim projekcijama (slika 1) i normalni aspekt u cilindričnim projekcijama (slika 2) zapravo dva različita aspekta bila bi opravdana kada bi se aspekt projekcije mogao jednoznačno definirati ovisno o tome je li u središtu karte geografski pol (polarni aspekt), točka na ekvatoru (ekvatorski aspekt) ili bilo koja točka između njih (kosi aspekt). Da takva definicija aspekta nije jednoznačno moguća pokazali smo u radu (Lapaine, Frančula 2019). Ovdje dajemo još jedan dokaz da se podjela aspekta na polarni, ekvatorski i kosi, osim na azimutne projekcije, ne može primijeniti na ostale skupine projekcija. Na slici 3 je karta SAD-a u normalnom aspektu konusne projekcije.

Ako aspekt dijelimo na polarni, ekvatorski i kosi (Kessler, Battersby 2019), pitanje je, u kojem je aspektu karta na slici 3. Budući da u središtu karte nije ni geografski pol ni ekvator, odgovor može biti jedino u kosom aspektu. Prema takvom načinu definiranja aspekta sve karte SAD-a su u kosom aspektu, što nije točno. Karte SAD-a, kao i karte većine drugih područja, mogu biti u sva tri aspekta: uspravnom, poprečnom i kosom. Na slici 3 je karta SAD-a u uspravnoj ekvivalentnoj konusnoj projekciji, a na slici 4 u poprečnoj konformnoj cilindričnoj projekciji. Za kartu bilo kojeg područja potrebno je prema njegovu položaju na Zemljinoj sferi i njegovim geometrijskim svojstvima (veličina, oblik, pružanje) te prema sadržaju i namjeni karte izabrati najprikladniju projekciju u najprikladnijem aspektu. Budući da se SAD prostire na srednjim širinama, da u smjeru sjever – jug nema veće pružanje od 32° i da mu je pružanje u smjeru zapad – istok veće od pružanja u smjeru sjever – jug, to su za karte SAD-a najpovoljnije uspravne konusne projekcije (Lapaine, Frančula 2005).

Da bismo potvrdili ispravnost takvog shvaćanja aspekta projekcije poslužit ćemo se citatima iz tekstova dobrih poznavalaca kartografskih projekcija. Snyder (1987) na str. 98 piše: „Like other normal conics, the Albers Equal-Area Conic projection (fig. 20) has concentric arcs of circles for parallels and equally spaced radii as meridians.“ To je oblik mreže meridijana i paralela i na slici 3 u ovom tekstu, dakle, riječ je o uspravnoj konusnoj projekciji.

Da se termini polarni i ekvatorski aspekt mogu primijeniti samo na azimutne projekcije Snyder (1987) objašnjava na str. 29: „If either true pole is at the center of an azimuthal map projection, the projection is called the *polar* aspect. If a point on the Equator is made the center, the projection is called the *equatorial* or, less often, *meridian* or *meridional* aspect. If some other point is central, the projection is the *oblique* or, occasionally, *horizon* aspect. For cylindrical and most other projections, such transformations are called *transverse* or *oblique*, depending on the angle of rotation.“

Canters (2002) u svoj knjizi o oblikovanju matematičke osnove karata sitnih mjerila na str. 73 piše; „Traditionally three aspects of a map projection are distinguished: the normal (or direct) aspect, the transverse aspect, and the oblique aspect“. U kazalu pojmova te knjige *aspect of map projection* u raznim svojim oblicima zauzima 27 redaka, a termina *polar aspect* i *equatorial aspect* nema.

Na kraju spomenimo i ovo. Normalni aspekt može se definirati i kao aspekt u kojem mreža meridijana i paralela ima u danoj skupini projekcija najjednostavniji oblik karakterističan za tu skupinu. Prema tome na slikama 1, 2, 3 i 4 nisu karte u polarnom, ekvatorskom i kosom aspektu nego karte u normalnom (polarnom) aspektu azimutnih projekcija (slika 1), normalnom aspektu cilindričnih projekcija (slika 2), normalnom aspektu konusnih projekcija (slika 3) i poprečnom aspektu cilindričnih projekcija (slika 4).

Kesslerova (2018) tvrdnja da normalni aspekt nije jednoznačno definiran ne stoji. Prema položaju osi projekcije u odnosu na os rotacije Zemljine sfere jednoznačno je definiran za sve važnije skupine projekcija. U normalnom aspektu os projekcije podudara se s osi rotacije Zemljine sfere (Frančula, Lapaine 2018). Jednoznačno je definiran i kao aspekt u kojemu mreža meridijana i paralela ima najjednostavniji oblik u danoj skupini projekcija.

I na kraju pitanje onima koji zagovaraju primjenu termina polarni, ekvatorski i kosi aspekt. U kojem je od ta tri aspekta poprečna Mercatorova projekcija koja uz mnoge ostale primjene čini i matematičku osnovu UTM-a?

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